A Level Set Approach to Image Segmentation With Intensity Inhomogeneity
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Abstract—It is often a difficult task to accurately segment images with intensity inhomogeneity, because most of representative algorithms are region-based that depend on intensity homogeneity of the interested object. In this paper, we present a novel level set method for image segmentation in the presence of intensity inhomogeneity. The inhomogeneous objects are modeled as Gaussian distributions of different means and variances in which a sliding window is used to map the original image into another domain, where the intensity distribution of each object is still Gaussian but better separated. The means of the Gaussian distributions in the transformed domain can be adaptively estimated by multiplying a bias field with the original signal within the window. A maximum likelihood energy functional is then defined on the whole image region, which combines the bias field, the level set function, and the piecewise constant function approximating the true image signal. The proposed level set method can be directly applied to simultaneous segmentation and bias correction for 3 and 7T magnetic resonance images. Extensive evaluation on synthetic and real-images demonstrate the superiority of the proposed method over other representative algorithms.

Index Terms—Active contour model, bias field correction, intensity inhomogeneity, level set method, segmentation.

I. INTRODUCTION

INTENSITY inhomogeneity caused by imperfection of imaging devices or illumination variations often occurs in real-world images, which leads to serious misclassification by intensity-based segmentation algorithms that assume a uniform intensity [1]–[6]. Statistically, misclassification is caused by the prolonged tails of intensity distribution of each object so that it is difficult to extract the desired objects accurately based on their respective intensity distributions. The well-known Mumford–Shah (MS) model [7], which assumes that the image intensity is piecewise smooth (PS), is suitable to model images with intensity inhomogeneity. The MS model uses a set of contours S to separate different regions. However, it is difficult to minimize its energy functional because the set S of low dimension is unknown and the problem is nonconvex [8]. Some simplified versions of the MS model have been proposed, such as the Chan–Vese (CV) model [1] and the PS model [8], which represent contour S as the zero level of a function called level set function, and then segmentation proceeds by evolving a level set equation. However, the CV model is not applicable to images with intensity inhomogeneity because it models images as piecewise constant functions. The PS model is able to achieve a desirable segmentation result for an image with intensity inhomogeneity. However, it needs to iterate two partial differential equations, which is very time-consuming and thereby limits its practical application.

Recently, some local region-based (LRB) level set methods have been proposed to deal with images with intensity inhomogeneity, such as LRB method [9], local binary fitting (LBF) model [10], local intensity clustering (LIC) method [11], local region model (LRM) [12], patch driven level set method [13] based on sparse representation [14]–[16], and edge driven level set method [17], etc. However, they have some drawbacks: the LRB method has two drawbacks. First, it used Dirac functional is restricted to a neighborhood around zero level set, which makes the level set evolution act locally. Therefore, the evolution can be easily trapped into local minima [1]. Second, its region descriptor is only based on region mean information without considering region variance and thereby may lead to inaccurate segmentation. This drawback also holds for the LBF model, which uses a similar energy functional. The LIC method can be considered as a locally weighted K-means clustering method [11], which does not consider the clustering variance; similar drawback often exists in the K-means clustering-based methods [18]. The LRM exploits local region statistics, i.e., local region means and variances, to interpret the MS model. However, the local region means and variances are only defined empirically, but not derived from minimizing the MS energy.

In this paper, we present a level set method for image segmentation. By exploiting the local image region statistics, we define a mapping from the original image domain to another domain in which intensity probability model is more robust to noise while suppressing the intensity overlapping to
some extent. We then devise a maximum likelihood energy functional based on the distribution of each local region in the transformed domain, which combines the bias field, the level set function, and the piecewise constant function approximating the true image signal. Analysis of the proposed approach shows that it is a soft classification method, which means that each pixel can be assigned to more than one class. In contrast, the hard classification [1] assigns each pixel to only one class [19]. In addition, the proposed method can be applied to simultaneous tissue segmentation and bias correction for magnetic resonance (MR) images.

The main differences between this paper and its preliminary presented in [20] are summarized as follows: 1) more related works and technique details about the principle of the proposed algorithm are provided such as the detailed discussions about how to optimize the proposed objective function are provided in Appendix A and B; 2) a new two-phase level set models are included; 3) the relationships among the proposed methods and other related works are discussed in detail; and 4) extensive evaluations on synthetic and real images are performed, including comparisons with the related works such as CV [1], global CV (GCV) [21], LBF [10], LCB [9], LBF [10], LIC [11] models.

The main contributions of this paper are summarized as follows.

1) It is difficult to use local region statistics to well segment images with severe intensity inhomogeneity because the regions must have sharp discontinuities in the statistics [22]. To handle this problem, we propose a very simple method by transforming the pixel intensities into another domain (via averaging the pixel intensity in a local region), in which we have theoretically validated that the intensities in the transformed domain have less overlapping in the statistics, thereby achieving better segmentation results than some representative level set methods [1], [9]–[11], [21] for images with severe intensity inhomogeneity.

2) The proposed method can yield the closed-form solutions for the estimated parameters in the distribution, which significantly reduces computation effort.

3) We show that some representative level set methods (e.g., CV [1], LBF [10], and LIC [11]) are the special cases of the proposed method.

### II. Notations

In the following sections, bold italic variables (e.g., \( \mathbf{x} \)) denote vectors; small letters (e.g., \( n \)) denote scalars; and capital letters (e.g., \( I \)) denote functions.

All notations are listed as follows: pixel coordinates \( \mathbf{x}, \mathbf{y} \in \Omega \subset \mathbb{R}^2 \), where \( \Omega \) represents image domain. The symbol \( \mathbb{R} \) denotes the set of real number and \( \emptyset \) is an empty set. The symbol \( I \) denotes an input image and \( \mathbf{I} \) denotes an image in the transformed domain. The symbols \( U, U_1, \) and \( U_2 \) are PS functions, respectively. The symbols \( \mu > 0, \nu > 0, \) and \( \rho \) are fixed parameters. Constant \( c_1 \) and \( c_2 \) approximate image intensities inside and outside the contour \( S \), respectively. The level set functions are denoted as \( \Phi, \Phi_1, \) and \( \Phi_2 \). The symbol \( \Phi \) is the set of level set function. \( H \) represents the Heaviside function and \( \delta \) denotes the Dirac function. \( B \) denotes bias field function and its optimal solution is denoted as \( \hat{B} \). The symbol \( J \) denotes true image signal. \( N \) represents Gaussian white noise. \( \sigma \) and \( \sigma_1 \) are standard deviations of the Gaussian distribution, respectively. The symbol \( T \) is the set of estimated parameters \( \{c_1, \sigma_1\} \), and their optimal solutions are denoted as \( \hat{c}_1, \hat{\sigma}_1 \). The symbol \( \Omega_k \) denotes a local region centering at location \( x \). The symbol \( L_i \) is the number of pixels. \( N \) denotes the Gaussian distribution. \( D \) sets the probability density function (PDF). \( D \) sets the set of image intensity in the transformed domain. \( K_p \) is an indicator function of a local region and \( G_p \) denotes a Gaussian function. \( M_i \) denotes the \( i \)th region indicator function. \( I \) denotes iterations. \( \Delta t \) and \( \Delta t_1 \) denote time steps, respectively. \( K \) denotes a kernel. \( E \) denotes an energy functional.

### III. Background

Mumford and Shah [7] approximated an image with a PS function \( U(\mathbf{x}) \), such that \( U \) varies smoothly within each sub-region, and abruptly across their boundaries. An energy functional is defined as [7]

\[
E_{U,S}^{MS} = \int_{\Omega \setminus S} (|dU|^2 + \mu |\nabla U|^2 + v|S|) \quad (1)
\]

where \( \mu > 0 \) and \( v > 0 \) are two fixed parameters and \( |S| \) represents the length of contour \( S \). Image segmentation can be performed by minimizing (1) with respect to \( U \) and \( S \). However, it is difficult to minimize \( E_{U,S}^{MS} \) in practice, due to the unknown set \( S \) of lower dimension and nonconvexity of the functional [8]. Many methods have been proposed to simplify or modify the functional [1], [8], which will be reviewed afterward. Here, we mainly discuss the two-phase case in which region \( \Omega \) is separated by a contour \( S \) where \( \Omega = \Omega_{in(S)} \cup \Omega_{out(S)} \) and \( \Omega_{in(S)} \cap \Omega_{out(S)} = \emptyset \).

Chan and Vese [1] proposed an active contour model that is a special case of (1) in which \( U(\mathbf{x}) \) in (1) is replaced by a piecewise-constant function. They proposed to minimize the following energy functional:

\[
E_{\Omega_1,\Omega_2}^{CV} = \int_{\Omega_{in(S)}} (|dU|^2 + \mu |\nabla U|^2 + v|S|) \quad (2)
\]

where \( c_1 \) and \( c_2 \) are two constant functions which approximate the average intensities inside and outside the contour \( S \), respectively. With the zero level set of the level set function \( \Phi \) to represent the contour \( S \), this model achieves favorable results for images with piecewise constant intensities.

The PS model [8] aims to minimize the energy functional (1) using the level set method. The contour \( S(p) : \mathbb{R} \rightarrow \Omega \) is implicitly represented by a level set function \( \Phi(\mathbf{x}) : \Omega \rightarrow \mathbb{R} \), i.e., \( S = \{ \mathbf{x} \in \Omega \mid \Phi(\mathbf{x}) = 0 \} \). By approximating an input image with two smooth functions \( U_1(\mathbf{x}) \) and \( U_2(\mathbf{x}) \) in the sub-regions \( \Omega_1 = \{ \mathbf{x} \in \Omega \mid \Phi(\mathbf{x}) > 0 \} \) and \( \Omega_2 = \{ \mathbf{x} \in \Omega \mid \Phi(\mathbf{x}) < 0 \} \), respectively, the energy functional of the PS model is...
i.e., assumed to be piecewise constant within each object domain, and the true signal to be restored, and the observed image $I(x)$.

**A. Statistical Model of Intensity Inhomogeneity**

The noise $N(x)$ is often assumed to be smooth in the image domain. Moreover, using only one Gaussian model is not accurate enough to describe the statistical characteristics of image intensity. Often multiple Gaussian probability distributions are adopted, with each one modeling the distribution of image intensity in each object domain. The distribution corresponding to the object domain $\Omega_i$ is [22]

$$P(I(y)|\theta_i, B, x) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp \left(-\frac{(I(y) - U_i(x))^2}{2\sigma_i^2}\right)$$  \hspace{1cm} (5)

where $U_i(x)$ is the spatially varying mean that is estimated at the local region $\Omega_i \cap O_x$ centered at each location $x$ (see Fig. 2), and $\sigma_i$ is the standard deviation. Since $B(x)$ varies slowly, it can be assumed to be a constant in a small window [11]. That is $U_i(x) \approx B(x)c_i$ for all $x \in \Omega_i \cap O_x$. We symbolize $\theta_i = \{c_i, \sigma_i\}$ and $\theta = \{\theta_i, i = 1, \ldots, n\}$ in the following discussions.

Using the local region statistics (5) is able to well describe the gradually varying regions that commonly exist in images with intensity inhomogeneity. However, the regions must have sharp discontinuities in the statistics for better separation [22]. In the following discussions, we propose a simple method to map the image intensity to another domain in which all regions have sharp discontinuities in the statistics. The main idea of our approach is to reduce the overlapping tails of the distribution in the transformed domain by compressing its profile with mean $U_i$ fixed [2] (see the red dashed curves in Fig. 3), thereby equipping it with better separation capability for all regions in the statistics.

**B. Principle of the Proposed Method**

As shown by Fig. 2, we denote $O_x$ a neighboring region centered at location $x$, i.e., $O_x = \{y|\|y-x\| \leq \rho\}$, where $\rho$ is the radius of the region $O_x$. The whole image domain $\Omega$ can be represented as $\Omega = \bigcup_{i=1}^n \Omega_i$ with $\Omega_i \cap \Omega_j = \emptyset$, for all $i \neq j$, where $\Omega_i$ is the $i$th object region. We define a mapping $T: I(x|\theta_i, B) \rightarrow \hat{I}(x|\theta_i, B)$ from original image intensity domain $D(T)$ to another domain $R(T)$ by averaging image intensities

$$\hat{I}(x|\theta_i, B) = \frac{1}{L_i(x)} \int_{\Omega_i \cap O_x} I(y|\theta_i, B, x) dy$$  \hspace{1cm} (6)

where $L_i(x) = |\Omega_i \cap O_x|$ is the number of pixels in region $\Omega_i \cap O_x$.
Remark: In this paper, (6) is not a preprocessing step of a test image because each region $\Omega_i$ is unknown at first, and our objective is to extract $\Omega_i$ by optimizing a statistical model based on $I$, which is computed with the statistical information of $I$ via their linkage in (6). This is significantly different from that uses a bilateral filter [24] on the test image as a preprocessing step (see comparison results in Section VI-C). Furthermore, the bilateral filter combines image intensities based on both their geometric closeness and photometric similarity, but the proposed method just averages the image intensities in a local region without considering their geometric or photometric similarity.

The intensity of pixel $x$ is assumed to be independently distributed with a Gaussian distribution [2], i.e., $I(y|\theta_i, B, x) = N(I(x|U_i(x), \sigma_i^2/L_i(x)))$, where $U_i$ is the spatially varying mean and $\sigma_i$ is the standard deviation subject to the object in region $\Omega_i$. Thus, for all $I(x|\theta_i) \in R(\mathcal{I})$, the corresponding PDF is still Gaussian [18] as

$$P(\mathcal{I}(x|\theta_i, B)) = N\left(I|U_i(x), \frac{\sigma_i^2}{L_i(x)}\right). \quad (7)$$

Referring to the red dashed curves in Fig. 3, the overlapping tails of the distributions are suppressed to some extent, thereby the pixel intensities in the transformed domain are much easier to be classified than that in the original domain.

In the following discussion, we first bridge the statistical linkage between the intensities $\mathcal{I}$ and $I$ via (9) because the PDF of $I$ is assumed to be known [see (5)]. Then, we build an objective function (11), which is the likelihood function by using the statistical information of $\mathcal{I}$. Finally, we use the log-likelihood function [see (15)] with the statistical information of $I$ as the total objective function.

Since the intensity inhomogeneity manifests itself as a smooth intensity variation across an image (see Fig. 1), similar to [11], we can reasonably assume that $I(y|\theta_i, B) \approx I(x|\theta_i, B)$, for all $y \in \Omega_i \cap O_x$. Thus, we have $\prod_{y \in \Omega_i \cap O_x} P(I(y|\theta_i, B)) \approx P(I(x|\theta_i, B)) \approx N(I(x|U_i(x), \sigma_i^2/L_i(x)))$. Therefore, we have

$$\prod_{y \in \Omega_i \cap O_x} P(I(y|\theta_i, B)) \approx N\left(I|U_i(x), \frac{\sigma_i^2}{L_i(x)}\right). \quad (8)$$

Putting (7) into (8), we have

$$P(\mathcal{I}(x|\theta_i, B)) \propto \prod_{y \in \Omega_i \cap O_x} P(I(y|\theta_i, B, x)) \quad (9)$$

which is the likelihood of $\theta_i$ with respect to the samples $\{I(y), y \in \Omega_i \cap O_x\}$.

Let $D = \{\mathcal{I}(x|\theta_i, B), x \in \Omega, i = 1, \ldots, n\}$, we have the following likelihood function for the $i$th object [18]:

$$P(D|\theta_i, B) = \prod_{x \in \Omega} P(\mathcal{I}(x|\theta_i, B)). \quad (10)$$

We have the following joint likelihood function:

$$P(D|\theta, B) = \prod_{i=1}^{n} P(D|\theta_i, B) = \prod_{i=1}^{n} \prod_{x \in \Omega} P(\mathcal{I}(x|\theta_i, B)) \quad (11)$$

$$\propto \prod_{i=1}^{n} \prod_{x \in \Omega} \prod_{y \in \Omega_i \cap O_x} P(I(y|\theta_i, B, x)),$$

where $\theta = \{\theta_i, i = 1, \ldots, n\}$.

Our objective is to estimate the parameter set $\tilde{\theta}$ and the bias field function $B$ that maximize the joint likelihood $P(D|\theta, B)$ in (11) and the likelihood $P(\mathcal{I}(x|\theta), B)$ in (9).

We define an energy functional $E^L(\theta, B)$ that is the inverse log-likelihood function of $P(D|\theta, B)$ in (11)

$$E_{\theta,B}^L \triangleq -\log P(D|\theta, B)$$

$$= \text{constant} - \sum_{i=1}^{n} \int_{\Omega} \int_{\Omega_i \cap O_x} \log(P(I(y|\theta_i, B, x)))dydx \quad (13)$$

Let $K_{\rho}(x, y)$ be the indicator function of region $O_x$ (see Fig. 2)

$$K_{\rho}(x, y) = \begin{cases} 1, & |y - x| \leq \rho \\ 0, & \text{else}. \end{cases} \quad (14)$$

Putting (5) and (14) into (13), and eliminating the trivial constant term, $E^L(\theta, B)$ can be rewritten as

$$E_{\theta,B}^L = \sum_{i=1}^{n} \int_{\Omega} \int_{\Omega_i \cap O_x} K_{\rho}(x, y) \left(\log(\sigma_i) + \frac{(I(y) - B(x)\sigma_i)^2}{2\sigma_i^2}\right)dydx. \quad (15)$$

C. Discussion

In this section, we discuss some advantages of the proposed method, including achieving a soft classification, robustness against noise, and alleviating side effect of oversmoothing object boundaries.

The joint likelihood function (11) can be rewritten as

$$P(D|\theta, B) = \prod_{x \in \Omega} Q(I(x|\theta, B)) \quad (16)$$

where

$$Q(I(x|\theta, B)) = \prod_{i=1}^{n} P(I(x|\theta_i, B)). \quad (17)$$

The joint likelihood function $P(D|\theta, B)$ in (16) can be viewed as the likelihood function of parameter set $\theta$ associated with the intensity $I$ whose PDF is $Q(I(x|\theta, B))$. 

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Put (8) and (9) into (17), $Q(I(x|\theta, B))$ can be rewritten as a univariate Gaussian distribution

$$Q(I(x|\theta, B)) = \prod_{i=1}^{n} P(I(x|\theta_i, B)) \propto \mathcal{N}(I|U(x), \Gamma(x))$$  \hspace{1cm} (18)

where

$$U(x) = \Gamma(x) \sum_{i=1}^{n} \frac{L_{i}(x)U_{i}(x)}{\sigma_{i}^2}, \Gamma^{-1}(x) = \sum_{i=1}^{n} \frac{L_{i}(x)}{\sigma_{i}^2}. \hspace{1cm} (19)$$

As shown by the PDF in (18) whose parameters are a combination of the parameters subject to all class intensities [see (19)], each pixel (voxel) intensity $I(x|\theta, B)$ is subject to multiple classes. Therefore, our method achieves a soft classification, satisfying the condition of the partial volume effect [25] (i.e., the intensity of each voxel is mixed from multiple classes [25]). Moreover, as shown from (6), the intensity in the transformed domain is obtained by averaging the neighboring pixel intensities belonging to the same class which is a low-pass filter process. Thus, the classification result is less sensitive to noise. Finally, our method alleviates the side effect of oversmoothing object boundaries due to the following reasons. First, only the neighboring intensities belonging to the same class contribute to each class of $I$ in (6); second, the overlapping parts of the statistical distributions among different classes of intensities are suppressed (see Fig. 3).

**D. Energy Functional Using the Level Set Method**

One level set function $\Phi$ can only represent two regions, inside and outside the contour $S$, as $\Omega_1 = \text{in}(S) = \{\Phi > 0\}$ and $\Omega_2 = \text{out}(S) = \{\Phi < 0\}$, respectively. This is called the two-phase model. If there exist more than two different regions, two level set functions, i.e., $\Phi_1$ and $\Phi_2$, can be used to represent all of the different regions based on the four-color theorem [8] such that any two adjacent regions can be indicated by different colors. This is called the four-phase model. We define the phase indicators as follows [8]:

- **Two phase:**
  \[ M_1(\Phi) = H(\Phi) \]
  \[ M_2(\Phi) = 1 - H(\Phi) \]
  \hspace{1cm} (20)

- **Four phase:**
  \[ M_1(\Phi) = H(\Phi_1)H(\Phi_2) \]
  \[ M_2(\Phi) = H(\Phi_1)(1 - H(\Phi_2)) \]
  \[ M_3(\Phi) = (1 - H(\Phi_1))H(\Phi_2) \]
  \[ M_4(\Phi) = (1 - H(\Phi_1))(1 - H(\Phi_2)) \]
  \hspace{1cm} (21)

where $H(\cdot)$ is the Heaviside functional and $\Phi$ represents the set of level set functions, i.e., $\Phi = \{\Phi\}$ for the two-phase model and $\Phi = \{\Phi_1, \Phi_2\}$ for the four-phase model. Each phase indicator function $M_i(\Phi)$ in (20) or (21) is the membership function of the region $\Omega_i$ satisfying

$$M_i(\Phi(y)) = \begin{cases} 1, y \in \Omega_i \\ 0, \text{else.} \end{cases} \hspace{1cm} (22)$$

Putting (22) into (15), the energy functional $E^L(\theta, B)$ is extended to the whole image domain

$$E^L_{\theta, B, \Phi} = \sum_{i=1}^{n} \int_{\Omega} F_i(y)M_i(\Phi(y))dy \hspace{1cm} (23)$$

where $F_i(y) \triangleq \int_{\Omega} K_{\rho}(x,y)(\log(\sigma_i) + ((I(y) - B(x)c_i)^2/2\sigma_i^2))dx$, and $n = 2$ or $4$, which constructs the force to drive the level set function to evolve. When the zero level set keeps stable, we obtain the final segmentation results in which each region is indicated by its membership function $M_i$ in (22).

**E. Relation to Other Models**

In this section, we explain the relationships of our method with six representative active contour models, i.e., the CV model [1], the PS model [8], the LRM method [12], the LBF model [10], the LIC model [11], and the Local Gaussian distribution (LGD) model [26] in detail.

We only discuss the two-phase level set method while similar discussions can be readily extended to the four-phase case. Let us revisit the proposed energy functional in (23). When the variance $\sigma_i^2 = 1, i = 1, 2$, and the bias field $B(x) = 1$, (23) can be rewritten as

$$E^L_{\theta, \Phi} = \frac{1}{2} \sum_{i=1}^{2} \int_{\Omega} \int_{\Omega} K_{\rho}(x,y)(I(y) - c_i)^2M_i(\Phi(y))dx dy$$

$$\quad = \frac{1}{2} \sum_{i=1}^{2} \left( \int_{\Omega} (I(y) - c_i)^2M_i(\Phi(y))dy \int_{\Omega} K_{\rho}(x,y)dx \right)$$

$$\quad = \frac{\pi \rho^2}{2} \sum_{i=1}^{2} \left( \int_{\Omega} (I(y) - c_i)^2M_i(\Phi(y))dy. \right) \hspace{1cm} (24)$$

The above energy functional is similar to that of the well-known CV model [1] except for the trivial constant $(\pi \rho^2/2)$ [see (2)].

In (23), if we assume that $U_i(x) = B(x)c_i$ is a PS function and $c_1 = 1$, the objective functional $E^L(\theta, B, \Phi)$ can be rewritten as that in (24), where $c_1$ and $c_2$ are replaced by two PS functions, respectively, which is similar to the objective functional of the PS model [see (3)] [8].

The LRM [12] handles inhomogeneous intensities well by using spatially varying means and variances to replace the constant means and variances in (2). However, the LRM method directly introduces a Gaussian kernel to compute the varying means and variances, which is inconsistent with theory [26].

The data-fitting term of the energy functional in the LBF model [10] is as follows:

$$E_{\Phi, U_1, U_2}^{\text{LBF}} = \int_{\Omega} \int_{\Omega} G_{\sigma}(y - x)(I(y) - U_1(x))^2H(\Phi(y))dx dy$$

$$\quad + \int_{\Omega} \int_{\Omega} G_{\sigma}(y - x)(I(y) - U_2(x))^2(1 - H(\Phi(y)))dx dy \hspace{1cm} (25)$$

where $G_{\sigma}(\cdot)$ is a truncated Gaussian kernel with standard deviation $\sigma$, which satisfies $\int_{\Omega} G_{\sigma}(x)dx = 1$, and $U_1(\cdot)$ and $U_2(\cdot)$ are two smoothing functions that approximate the weighted average image intensities in a Gaussian window in the regions $\Omega_1 = \{\Phi(x) > 0\}$ and $\Omega_2 = \{\Phi(x) < 0\}$, respectively.

It is worth noting that the principles of the proposed model, the LBF model [10] and LIC model [11] are different. If we set $\sigma_i^2 = 1, B(x)c_i = U_i(x)$ in (23), and use a truncated Gaussian kernel $K_{\rho}$ with standard deviation $\rho$ satisfying...
\[ \int_{\Omega} K_{\rho}(x)dx = 1 \] to replace the constant kernel, then (23) can be written as
\[
E_{U_{1}, U_{2}}^{L} = \frac{1}{2} \int_{\Omega} \int_{\Omega} K_{\rho}(y-x)(I(y) - U_{1}(x))^{2}H(\Phi(y))dxdy \\
+ \frac{1}{2} \int_{\Omega} \int_{\Omega} K_{\rho}(y-x)(I(y) - U_{2}(x))^{2}(1 - H(\Phi(y)))dxdy.
\tag{26}
\]

This is the same as \( E_{\Phi, U_{1}, U_{2}}^{LBF} \) except for the trivial constant 1/2. If we set \( U_{1} = Bc_{i} \) in (26), it is the energy functional of the LIC model [11]. It should be noted that in the theoretical analysis of our method in Section IV-B, the constant kernel is reasonably used as a local region indicator. However, both of the LBF and LIC models use a Gaussian kernel as the locally spatially weighted function to relate the pixel \( x \) and its neighboring pixel \( y \). The closer \( y \) is to \( x \), the larger weight is assigned, thereby representing the higher similarity between the intensities of pixels \( y \) and \( x \).

The energy functional of the LGD model [26] is slightly different from the LBF model which replaces the \( \ell_{2} \)-norm terms in (26) with \( ((I(y) - U_{i}(x))^{2}/\sigma_{i}^{2}(x)) \), \( i = 1, 2 \), where \( U_{i}(x) \) and \( \sigma_{i}(x) \) are spatially locally varying mean and variance of a Gaussian distribution. However, as pointed out by [22], the spatially varying variance may be unstable due to its local property. Our model is different from the LGD model due to the following reasons. First, the variances of the Gaussian distributions in our model are piecewise constant in each region, thereby making our method much more stable than the LGD model; second, we use a constant kernel to indicate a local region which is based on a solid theoretical foundation as explained in Section IV-B. However, the Gaussian kernel is used in LGD model whose physical meaning is similar to the LBF model; third, our model can be used for simultaneous segmentation and bias correction while the LGD model can be only used for segmentation.

V. Energy Minimization and Level Set Evolution Formulas

Minimizing \( E_{\theta, B, \Phi}^{L} \) with respect to each variable in \( \theta = \{ c_{i}, \sigma_{i}, i = 1, \ldots, n \} \) and \( B \), we can obtain their closed-form solutions, which are described below.

A. Closed-Form Solutions for Different Variables

1) Minimization With Respect to \( c_{i} \): By fixing the other variables in (23), we obtain the minimizer of \( c_{i} \), denoted by \( \tilde{c}_{i} \), as follows:
\[
\tilde{c}_{i} = \frac{\int_{\Omega} (K_{\rho} \otimes B)M_{i}(\Phi(y))dy}{\int_{\Omega} (K_{\rho} \otimes B^{2})M_{i}(\Phi(y))dy}
\tag{27}
\]
where \( \otimes \) denotes the convolution operator.

2) Minimization With Respect to \( B \): By fixing the other variables in (23), we obtain the minimizer of \( B \), denoted by \( \tilde{B} \), as follows:
\[
\tilde{B}(x) = \frac{\sum_{i=1}^{n} K_{\rho} \otimes (IM_{i}(\Phi(x))) \cdot \frac{c_{i}^{2}}{\sigma_{i}^{4}}}{\sum_{i=1}^{n} K_{\rho} \otimes M_{i}(\Phi(x)) \cdot \frac{c_{i}^{2}}{\sigma_{i}^{4}}}
\tag{28}
\]

Note that \( \tilde{B} \) is actually the normalized convolution [27], which naturally leads to a smooth approximation of the bias field \( B \).

3) Minimization With Respect to \( \sigma_{i} \): By fixing the other variables in (23), we can obtain the minimizer of \( \sigma_{i} \), denoted by \( \bar{\sigma}_{i} \), as follows:
\[
\bar{\sigma}_{i} = \left( \frac{\int_{\Omega} \int_{\Omega} K_{\rho}(y-x)M_{i}(\Phi(y))(I(y) - B(x)c_{i})^{2}dydx}{\Omega \Omega} \right)^{1/2}
\tag{29}
\]
For an explanation of how to derive above solutions, please refer to Appendixes A and B.

The solutions \( \theta = \{ \tilde{c}_{i}, \bar{\sigma}_{i}, i = 1, \ldots, n, n = 2, \text{ or } 4 \} \) and \( \tilde{B} \) are then put into (23) in which the solution of \( F_{i} \) is
\[
\tilde{F}_{i}(y) = \int_{\Omega} \int_{\Omega} K_{\rho}(y-x)(\log(\bar{\sigma}_{i}) + \frac{(I(y) - \tilde{B}(x)\tilde{c}_{i})^{2}}{2\bar{\sigma}_{i}^{2}})dx.
\tag{30}
\]

B. Two-Phase Level Set Evolution Formula

Minimizing the energy functional \( E_{\theta, B, \Phi}^{L} \) with respect to \( \Phi \), we have the corresponding gradient descent formula as follows:
\[
\frac{\partial \Phi}{\partial t} = - \frac{\partial E_{\theta, B, \Phi}(\Phi)}{\partial \Phi} = (\tilde{F}_{2} - \tilde{F}_{1})\delta(\Phi)
\tag{31}
\]
where \( \delta(\Phi) \) is the Dirac functional.

In order to keep the numerical implementation stable, the level set function should be regularized during the iteration of (31). Li et al. [28] proposed a signed distance regularization formula. However, it produces some unnecessary valleys and peaks, which makes level set evolution easy to fall into some local minima. In this paper, we adopt a simple and stable method [29] to regularize the level set function during iteration. After each iteration of the level set evolution, we diffuse the level set function using the following formula:
\[
\Phi^{i+1} = \Phi^{i} + \Delta t \cdot \nabla^{2} \Phi^{i}
\tag{32}
\]
where \( \Phi^{i} \) represents the level set function obtained from the \( i \)-th iteration of (31), \( \nabla^{2} \) represents the Laplacian operator, and \( \Delta t \) represents the diffusion strength. The \( \Phi^{i+1} \) in (32) can also be approximated by \( \Phi^{i+1} = K \otimes \Phi^{i} \), where \( K \) is either a Gaussian kernel [30] or a constant kernel [20].

C. Four-Phase Level Set Evolution Formula

Minimizing the energy functional \( E_{\theta, B, \Phi}^{L} \) with respect to \( \Phi_{1} \) and \( \Phi_{2} \), respectively, we have the corresponding gradient-descent formulas
\[
\begin{aligned}
\frac{\partial \Phi_{1}}{\partial t} &= -((\tilde{F}_{1} - \tilde{F}_{2} - \tilde{F}_{3} + \tilde{F}_{4})H(\Phi_{2}) + \tilde{F}_{2} - \tilde{F}_{4})\delta(\Phi_{1}) \\
\frac{\partial \Phi_{2}}{\partial t} &= -((\tilde{F}_{1} - \tilde{F}_{2} - \tilde{F}_{3} + \tilde{F}_{4})H(\Phi_{1}) + \tilde{F}_{3} - \tilde{F}_{4})\delta(\Phi_{2}).
\end{aligned}
\tag{33}
\]

Each level set function is regularized by the following formula after each iteration of (33):
\[
\Phi_{i}^{i+1} = \Phi_{i}^{i} + \Delta t \cdot \nabla^{2} \Phi_{i}^{i}, i = 1, 2.
\tag{34}
\]
Algorithm 1 Locally Statistical Level Set Method

1: Initialization: \( \tilde{B} = 1, \tilde{\sigma}_i = i \) (\( i \) is taken as both variable value and index), \( i = 1, \ldots, n \), \( n = 2, 4, \) and the level set function \( \Phi^{(l)} = \Phi^{(l)}_i, i = 1, \ldots, n \)
2: Update \( c_i \) to \( \tilde{c}_i, i = 1, \ldots, n \), by (27)
3: Update \( B \) to \( \tilde{B} \) by (28)
4: Update \( \sigma_i \) to \( \tilde{\sigma}_i, i = 1, \ldots, n \), by (29)
5: Update \( F_j \) to \( \tilde{F}_j, i = 1, \ldots, n \) by (30)
6: Evolve the level set function according to (32) or (34) once
7: Regularize the level set function according to (35)
8: If \( \Phi^{(l+1)} \) satisfies the stationary condition, stop; otherwise, \( l = l + 1 \) and return to Step 2.

D. Numerical Implementation

We only need to approximate the temporal derivatives in (32) and (33) as a forward difference because there are no partial derivatives. The Laplacian operator \( \nabla^2 \) is approximated as \( \nabla^2 \Phi \approx K \otimes \Phi \), where \( \otimes \) is a convolution operator, and \( K \) is a kernel defined as
\[
K = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}.
\]
Therefore, the solution of the diffusion equation (32) can be discretized as follows:
\[
\Phi^{(l+1)} = \Phi^{(l)} \otimes \begin{pmatrix} 0 & \Delta t & 0 \\ \Delta t & 1 - 4\Delta t & \Delta t \\ 0 & \Delta t & 0 \end{pmatrix}. \tag{35}
\]

The standard Von Neumann analysis [29] can be used to analyze the stability for the time step \( \Delta t \). Putting \( \Phi(i,j) = r^i e^{i\xi_1} e^{j\xi_2} \) into (35), where \( \sqrt{\xi_1^2 + \xi_2^2} = 1 \) denotes the imaginary unit, we obtain the amplification factor as follows:
\[
r = 1 + 2\Delta t \cdot (\cos(\xi_1) + \cos(\xi_2) - 2). \tag{36}
\]
Therefore, we have \( 1 - 8\Delta t \leq r \leq 1 \). By solving the inequality \( |1-8\Delta t| \leq 1 \), we have
\[
0 \leq \Delta t \leq 0.25. \tag{37}
\]

The Heaviside functional \( H(z) \) is approximated by a smooth function \( H_\varepsilon(z) \) as
\[
H_\varepsilon(z) = \frac{1}{2} \left( 1 + \frac{2}{\pi} \arctan \left( \frac{z}{\varepsilon} \right) \right), z \in \mathbb{R} \tag{38}
\]
where \( \varepsilon > 0 \) is a constant. The Dirac functional \( \delta(z) \) is approximated by \( \delta_\varepsilon(z) \) as
\[
\delta_\varepsilon(z) = \frac{d H_\varepsilon(z)}{dz} = \frac{1}{\pi \varepsilon^2 + z^2}, z \in \mathbb{R}. \tag{39}
\]

The procedures of the proposed algorithm are summarized in Algorithm 1.

VI. Experimental Results

In this section, we compare our method with the CV model [1], the GCV model [21], the LR model [9], the LIC model [11], and the LBF model [10], which are some representative level set methods for image segmentation. The MATLAB source codes and some examples of the proposed method can be downloaded at http://www.comp.polyu.edu.hk/~cszhang/LSACM/LSACM.htm.

We initialize \( \tilde{B}(x) = 1, \tilde{\sigma}_i = i, i = 1, \ldots, n \), and then the initializations of \( \tilde{c}_i, i = 1, \ldots, n \) can be calculated by (27). We have tested different values to initialize \( \tilde{B}(x) \) and \( \tilde{c}_i \), and found that their corresponding results are similar, which can be attributed to the convexity of our objective functional (23) with respect to these variables. The time step for level set evolution is set \( \Delta t_1 = 1 \), the time step for regularization is set \( \Delta t = 0.1 \), and \( \varepsilon = 1 \) for all the experiments except for Figs. 11–13, in which we set \( \Delta t = 0.01 \). Our method is stable for a wide range of \( \rho \), e.g., \( 5 < \rho < 25 \). In most cases, we set \( \rho = 6 \). A small \( \rho \) makes computation in each iteration more efficient, but the convergence of the algorithm is slower. On the other hand, a large \( \rho \) increases computational burden in each iteration. However, the convergence rate can be increased because information from larger regions is exploited. Therefore, the total computation burden is comparable for different \( \rho \).

A. Comparisons With the CV [1] and GCV [21] Models

Both the CV [1] and GCV [21] models assume that image intensities are piecewise constant and thereby use global intensity means to fit them. Therefore, they do not perform well on images with intensity inhomogeneity. In this section, we compare our method with the CV and GCV models on some real and synthetic images with intensity inhomogeneity.

Fig. 4 demonstrates the segmentation results on two synthetic images and two real-vessel images with intensity inhomogeneity. From top to bottom: segmentation results by the CV model [1], the GCV model [21], and our method. The red circles represent the initial contours and the red lines represent the final segmentation contours. We set \( \rho = 12 \) for the left image and \( \rho = 6 \) for the other images.
region information and thereby result in severe misclassification on these images because there exist severe overlapping intensities between each object and its background.

B. Results on Synthetic and Real Images

We first use one synthetic image and two real-MR images with severe intensity inhomogeneity to demonstrate the superior performance of our method to the LBF model, the LIC model and the LRB model, all of which use local means to fit image intensities and thereby perform well on images with light intensity inhomogeneity. The LBF model [10] uses the local intensity mean to fit the intensities of the measured images and thereby achieves much better segmentation results than the CV and GCV models on images with intensity inhomogeneity. However, if the intensity inhomogeneity is severe, using only the local mean information may fail to discriminate the intensities of an object from its background, thereby leading to inaccurate segmentation. Some experimental results are shown in the second column of Fig. 5. Similar drawbacks exist for the LIC model [11] (see the third column of Fig. 5) and LRB model [9]. Moreover, as shown in the fourth column of Fig. 5, the localized Heaviside and Dirac functionals adopted by the LRB model make the level set evolution easily fall into local minima [1]. Our method can produce much better segmentation results because it takes into account the statistical information of intensities in a transformed domain in which the intensities of object and background are less overlapping than those in the original domain. Then, we test our method on three natural images with complicate textures (see Fig. 6). Our method can achieve satisfying segmentation results on them, which demonstrates its favorable generality.

C. Results on Images Preprocessed by Bilateral Filtering

In Fig. 7, we evaluate the performance of the competing methods on five synthetic and real-images selected from Figs. 4 and 5 in which our method achieves favorable results. The test images are first preprocessed by bilateral filtering [24] (see the top row of Fig. 7). Compared with their original images in Figs. 4 and 5, it is obvious that intensity contrast between object and background is more obvious after preprocessing, which in turn results in an easier separation of the object from background. Therefore, for the three real images, the LBF, LIC and LRB methods achieve better results on the preprocessed images than the original ones. However, some small noisy objects also have been extracted, thereby making the results less accurate than our method. Furthermore, the intensities of the preprocessed images are still inhomogeneous and thereby the CV and GCV methods cannot work well on them (see the second and third rows of Fig. 7) because both the CV and GCV methods are only suitable to segment images with piecewise constant intensities.

D. Application to Simultaneous Segmentation and Bias Correction

In this section, we apply our level set method for simultaneous segmentation and bias correction, especially for MR images. We compare our method with the LIC model [11] because only the LIC model is applicable for simultaneous segmentation and bias correction while the other competing models can only be used for segmentation.

Fig. 8 shows the results by the two competing methods on synthetic images corrupted with additive Gaussian noises of
Fig. 8. Experiments on a synthetic image with different additive Gaussian noises. The test images are corrupted with Gaussian white noises of standard deviations $\sigma = 1$ and $\sigma = 5$, respectively. From left to right: results by our method, the LIC model [11], the ground-truth bias field, the estimated bias field using our method, and the estimated bias field using the LIC model [11]. We set $\rho = 10$ for the experiments.

Fig. 9. From left to right: the initializations of level set functions $\Phi_1$ and $\Phi_2$ (the red lines represent the initial zero level sets of $\Phi_1$, while the blue lines represent the initial zero level sets of $\Phi_2$), estimated bias fields, tissue-classification results, and bias corrected images. The first and third rows are the results by our method with two different initializations, while the second and fourth rows are the results by the LIC model [11] with the same initializations. We set $\rho = 10$ for all the experiments.

different levels. The evaluation criteria are the final segmentation results and the similarity between the estimated bias field and the ground-truth bias. The top-left image in Fig. 8 was added with Gaussian white noise of zero mean and unit standard deviation. Since the noise level is low, both models yield satisfying segmentation results, while our method outperforms a little the LIC model. However, with increasing the noise level, the segmentation results by the LIC method are becoming very noisy (see bottom-left second figure in Fig. 8), whereas the segmentation result using our method is much better. This is because our model takes into account the different probability distributions of various objects. Furthermore, the estimated bias fields by our method are visually much more similar to the ground truth than the LIC method (see the right two columns of Fig. 8).

Fig. 9 shows the joint segmentation and bias correction results on a 3T MRI image, which has four classes of tissues: 1) whiter matter; 2) gray matter; 3) cerebrospinal fluid; and 4) background. Because one level set function can only represent two classes of tissues, we need to evolve two level set functions $\Phi_1$ and $\Phi_2$ according to (33) to represent four classes of tissues [8]. It can be seen that the tissue segmentation results by our method (the first and third rows in Fig. 9) are much more accurate than those by the LIC method (the second and fourth rows in Fig. 9). It should be noted that it is very easy to initialize the level set functions in our method in which the initial contours can be set inside, outside or across the object boundary. The two initializations in the first and third rows are very different, but the final segmentation results are very similar, which demonstrates the robustness of our method to different initializations.

Finally, Fig. 10 shows the bias-correction results using our method on two 7T MRI images. This experiment aims to obtain the bias corrected MRI images. The original images, estimated bias fields, and the bias-corrected images are shown in the left, middle, and right columns, respectively. It can be clearly seen that the image quality is significantly improved by our method. Some regions (inside the red circles) whose intensity contrast is too low to be identified are able to be distinguished clearly after the bias correction.

E. Quantitative Evaluation

In this section, we use Jaccard similarity (JS) [31], [32] to quantitatively evaluate the performance of the competing methods on images whose intensity inhomogeneity has different strength. The JS index between two regions $S_1$ and $S_2$ is calculated as $\text{JS}(S_1, S_2) = |S_1 \cap S_2| / |S_1 \cup S_2|$, which is the ratio between the intersectional area of $S_1$ and $S_2$ and their union. Obviously, the closer the JS value is to 1, the more similar $S_1$ is to $S_2$. In our experiments, $S_1$ is the segmented object region, and $S_2$ is the ground-truth.
and Different Types of Noises

We test the competing methods on five synthetic images with different intensity inhomogeneity. Their corresponding JS values are shown in the bottom image of Fig. 11. Obviously, the JS values obtained by our method have little difference for intensity inhomogeneity with different strength, which demonstrates that our method is very robust to image intensity inhomogeneity. For the CV and GCV models, when the strength of intensity inhomogeneity is not strong (see images 1 and 2 in Fig. 11), both of them can yield a high JS value. However, when the strength of inhomogeneity is not low (refer to images 3–5), the performance of the CV and GCV models degenerates rapidly. For the LRB models (i.e., the LRB model, the LBF model and the LIC model), when the strength of intensity inhomogeneity is strong (refer to images 4 and 5), the performance of these methods also degenerates severely. This is because only using local region means cannot discriminate object from its background satisfactorily when the intensities between them overlaps severely. Our method consistently yields the highest JS values because it pursues segmentation in a transformed domain where the overlapping intensities are effectively suppressed. The final segmentation results for the top right-most image in Fig. 11 by the six competing methods are shown in Fig. 12. The right-most segmentation results are generated by canny edge detector [33] (available from the MATLAB command edge). We set $\rho = 10.5$ for all the experiments.

We use the test image in Fig. 12 to demonstrate the robustness of our method to different level set initializations and region scale parameters $\rho$. We again use the JS index to measure segmentation accuracy. The top row in Fig. 13 demonstrates the segmentation results using different initial contours (the blue lines). We can see that there are no obvious visual differences for these segmentation results (the JS values of these results correspond to the first six values shown in the bottom-left figure of Fig. 12). We adopt 20 different initial contours (the region scale parameter is set $\rho = 10.5$). From the bottom-left figure of Fig. 13, we can observe that the JS values change only from 0.97 to 0.98, which clearly demonstrate that our method can yield very high and stable segmentation accuracies with different level set initializations.

The bottom-right figure of Fig. 13 shows the JS values for 20 different initial contours (the first six values are for the six segmentations in the top row), while the right figure shows the JS values for different region scale parameters $\rho$. The initial contour is the same as that in the top right-most image.

F. Robustness to Initializations, Region Scale Parameters, and Different Types of Noises

We use the test image in Fig. 12 to demonstrate the robustness of our method to different level set initializations and region scale parameters $\rho$. We again use the JS index to measure segmentation accuracy. The top row in Fig. 13 demonstrates the segmentation results using different initial contours (the blue lines). We can see that there are no obvious visual differences for these segmentation results (the JS values of these results correspond to the first six values shown in the bottom-left figure of Fig. 12). We adopt 20 different initial contours (the region scale parameter is set $\rho = 10.5$). From the bottom-left figure of Fig. 13, we can observe that the JS values change only from 0.97 to 0.98, which clearly demonstrate that our method can yield very high and stable segmentation accuracies with different level set initializations. The bottom-right figure of Fig. 13 shows the JS values computed by changing the region scale parameters $\rho$ from 5.5 to 22.5. The high and stable JS values again demonstrate that our method is very robust to the region scale parameters in a wide range.
Fig. 14 illustrates the segmentation results for images with different types of noises. The images on the top row of Fig. 14 are with different levels of Gaussian white noises, while the images on the bottom row are with different levels of multiplicative noises. We can observe that segmentation results are visually satisfying although the noises are very strong, thereby validating the robust performance of our method to different types of noises. Furthermore, as shown by Fig. 15, the JS values on images with multiplicative noises are a little smaller than those with Gaussian white noises, which can be attributed to the assumption of Gaussian distribution in our method. However, the differences (changes from 0.973 to 0.968) of JS values are so small that the visual effects of the segmentation results in Fig. 14 are not obvious.

**VII. CONCLUSION**

In this paper, by exploiting local image region statistics, we present a level set method for segmenting images with intensity inhomogeneity. Our method combines the information from the neighboring pixels belonging to the same class, which equips it with a strong capability to separate the desired object from its background. Moreover, the proposed method yields a soft classification, which can, to some extent, satisfy the condition of the partial volume effect. In addition, the segmentation results are insensitive to different initializations of the level set function, making it useful for automatic applications. Comparisons with several representative methods on synthetic and real images have demonstrated the effectiveness of the proposed algorithm.

**APPENDIX A**

In (23), we assume that the optimal solution of $c_i$ is $\tilde{c}_i$, and then we add a variation $\eta_i$ to the variable $\tilde{c}_i$ such that $c_i = \tilde{c}_i + \epsilon \eta_i$. Keeping other variables except for $c_i$ fixed, differentiating $E^L$ with respect to $c_i$ and letting $\epsilon \to 0^+$, we have

$$\frac{\delta E^L}{\delta \tilde{c}_i} = \lim_{\epsilon \to 0^+} \frac{dE^L}{d\epsilon} = -\eta_i \int \int_{\Omega \Omega} K_\rho(x,y) \frac{(I(y)B(x) - B^2(x)\tilde{c}_i)}{\sigma_i^2} M_i(\Phi(y)) dy dx = 0. \quad (40)$$

Therefore, we have

$$\int \int_{\Omega \Omega} K_\rho(x,y) \frac{(I(y)B(x) - B^2(x)\tilde{c}_i)}{\sigma_i^2} M_i(\Phi(y)) dy dx = 0. \quad (41)$$

From the above equation, we obtain

$$\tilde{c}_i = \frac{\int_{\Omega} (K_\rho \odot B)M_i(\Phi(y)) dy}{\int_{\Omega} (K_\rho \odot B^2)M_i(\Phi(y)) dy} \quad (42)$$

which is (27).

The $\tilde{\sigma}_i$ in (29) can be obtained using a similar method.

**APPENDIX B**

In (23), we assume that the optimal smoothing function $B(x)$ is $\tilde{B}(x)$. Then, we add a variation function $\eta(x)$ such that $B(x) = \tilde{B}(x) + \epsilon \eta(x)$. Differentiating $E^L$ with respect to $B$ and letting $\epsilon \to 0^+$, we have

$$\frac{\delta E^L}{\delta B} = \lim_{\epsilon \to 0^+} \frac{dE^L}{d\epsilon} = \int \int_{\Omega} K_\rho(x,y) \frac{(I(y) - \tilde{B}(x)\tilde{c}_i)}{\sigma_i^2} c_i dy \eta(x) dx. \quad (43)$$

Finally, we obtain

$$\tilde{B}(x) = \frac{\sum_{i=1}^{n} K_\rho \odot (IM_i(\Phi(x))) \cdot \tilde{c}_i}{\sum_{i=1}^{n} K_\rho \odot M_i(\Phi(x)) \cdot \tilde{c}_i^2} \quad (44)$$

which is (28).

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